The Josephson Effect and e/h

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A Josephson junction, consisting of two superconductors weakly coupled together, is an example of a very simple macroscopic quantum system. When there is a voltage V across the junction, the supercurrents flowing through it oscillate at a frequency given very precisely by the relation $\nu = 2eV/h$. The general validity of this formula has been used recently by Parker, Langenberg, Denenstein, and Taylor as the basis for a new determination of the fundamental constant ratio e/h. The new value is 38 ± 10 ppm smaller than the previously accepted value. This result has far-reaching implications concerning the values of the fundamental constants and quantum electrodynamics. This article begins with a simple derivation of the Josephson voltage–frequency relation. Theory suggests that there should be no significant corrections to this equation, a result which is well confirmed by experiment. Various types of junction are then described, together with their applications to the measurement of e/h. The impact of this measurement upon the fundamental constants and quantum electrodynamics is briefly discussed. It seems likely that Josephson junctions will have an important application in comparing and maintaining standards of electromotive force. [*This is another in the series of review-tutorial papers that are written at the invitation of the Editor and that will be published from time to time in the American Journal of Physics. Articles in this series will be of an expository, critical nature and are intended to be readable by students and professionals alike as a first introduction to a specialized topic.*]

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I. INTRODUCTION

At first sight, it may seem remarkable that a superconductor containing perhaps 10^{22} electrons/cm^3 may be used in a high-precision measurement of a fundamental constant involving the charge on a single electron. That this measurement is, in

fact, possible is a direct consequence of the macroscopic quantum state which exists in a superconductor.

The concept of a highly correlated condensed state in a superconductor was first suggested by London[1] and is central to the modern theory of superconductivity developed by Bardeen, Cooper, and Schrieffer.[2] A striking illustration of this quantum state is the quantization of the flux[1,3,4] contained in a superconducting ring into units of $hc/2e$, the flux quantum (h is Planck's constant, c the velocity of light, and e the electronic charge). Another important example is the junction composed of two superconductors which are weakly connected together. This system was first studied theoretically by Josephson.[5] Josephson junctions have subsequently been used in a number of devices,[6] for example, for the measurement of very tiny magnetic fields[7] and voltages,[8] the detection[9] and generation[10] of microwave and submillimeter radiation, and the determination of e/h.[11]

One of Josephson's several predictions[5] was that if a steady potential difference V was maintained across two weakly coupled superconductors, a supercurrent which oscillated at a frequency $\nu = 2eV/h$ would flow between them. The presence of this alternating supercurrent has been amply verified by experiment. It is clear that the simultaneous measurement of the voltage V and the corresponding frequency ν will enable the quantity $2e/h$ to be determined. Parker, Langenberg, Denenstein, and Taylor[11] have performed a series of high-precision experiments from which they deduced a value of $2e/h$. This value was 38 ± 10 ppm smaller than that calculated from the fundamental constants given by Cohen and Du Mond[12] in their 1963 least-squares adjustment.

The purpose of this article is to review the present status of the new e/h measurements and to discuss their implications. In Sec. II we outline the theoretical basis of the Josephson effect and show how it relates the fundamental quantities e and h. Section II is also concerned with types of Josephson junctions, simple experiments upon them, and possible methods of using the effect to determine e/h. Section III describes the measurements of e/h which have been made and gives the numerical results. In Sec. IV we are concerned with the absolute accuracy of the Josephson relation $\nu = 2eV/h$. We discuss a number of

experiments which suggest that the equation is valid to a high level of accuracy and also mention various theoretical considerations which indicate that there are no known deviations from the equation. Sections V and VI describe the applications and implications of the new measurement. The proposed use of Josephson junctions as a tool for maintaining and comparing standards of electromotive force is described in Sec. V. Section VI briefly outlines the role of the e/h measurement in the determination of fundamental constants. Since the Josephson value of e/h is believed to contain no quantum electrodynamic corrections, it is useful to compare the values of quantities derived from it (for example, α, the fine structure constant) with values deduced from methods which rely partly or wholly on quantum electrodynamics. Section VI also contains the results of this comparison. Section VII summarizes the conclusions.

The reader does not require a detailed knowledge of the theory of superconductivity. For an amplification of the concepts mentioned very briefly at the beginning of Sec. II, he is referred to any of the several texts available.[13] Excellent, comprehensive reviews of the Josephson effect have been given by Josephson[14] and Anderson.[15]

II. THE JOSEPHSON EFFECT

II.1. Theory

When a metal becomes superconducting at low temperatures, some of the free electrons become paired together.[2,16] The electron–electron attraction producing this pairing arises from the electron–phonon interaction, which is required to be greater than the Coulomb repulsion between the electrons in order for the metal to become superconducting. This electron–phonon interaction is of short range, on the order of an atomic spacing. On the other hand, the spatial extent of the paired electrons is much larger, on the order of a coherence length ξ, typically 10^{-5} cm. A sphere of diameter ξ contains roughly 10^6–10^7 pairs. In order for the pairs to take the best advantage of the attractive electron–phonon interaction, they all must have identical center-of-mass momenta. More generally, if we assign a phase factor to the wave function describing each pair, all of the phases of the pairs must be the same. If we

consider a large number of overlapping spheres, in each of which the phases of the pairs are the same, we can see that this basic *phase coherence* will extend throughout the whole superconductor. Thus, if we fix the phase at any one point in a bulk superconductor, the relative phase at all other points will be established in space and time. As is usual with a quantum mechanical wave function, we can add an arbitrary uniform phase factor to the whole superconductor without producing any new physical effects.

The idea of paired electrons was first proposed by Cooper[16] and subsequently developed by Bardeen, Cooper, and Schrieffer[2] into the microscopic theory of superconductivity. The idea of phase coherence is central to the whole theory and is ultimately responsible for the resistanceless current flow,[17] Meissner[18] effect, and flux quantization[1] which typify superconductors. The superconducting state with paired electrons is lower in energy than the corresponding normal state by an amount known as the "condensation energy." This energy is tiny, perhaps 10^{-8} eV/atom, and yet, it is sufficient to enforce the long-range phase coherence of the superconductor. The pairs are in a condensed state which may be described by a single macroscopic wave function. It is for this reason that we can study on a *macroscopic* scale quantum effects which have their origins in *microscopic* quantum processes.

In their phenomenological description of superconductivity, which preceeded the microscopic theory, Ginzburg and Landau[19] introduced a complex order parameter $\psi(\mathbf{r}, t) = |\psi(\mathbf{r}, t)| \exp[i\phi(\mathbf{r}, t)]$ as the wave function of the condensed state. The density of pairs in the superconductor is given by $|\psi(\mathbf{r}, t)|^2$ and the phase of the condensate by $\phi(\mathbf{r}, t)$. We emphasize that the whole condensate is to be described by this single wave function $\psi(\mathbf{r}, t)$; the phase $\phi(\mathbf{r}, t)$ is not the phase of a single electron or pair of electrons but rather that of all the pairs. The phase is consequently a physical observable.

Let us turn now to the basic ideas of the Josephson effect. If we have two superconductors completely isolated from one another, it is clearly possible to vary the phase ϕ of each superconductor separately.[14] On the other hand, as we have just seen, the phase in a single superconductor is determined everywhere apart from an arbitrary

FIG. 1. (a) In a bulk superconductor, in the absence of applied fields and currents, the phase of the wave function, ϕ, is constant; (b) For two isolated superconductors, the phase of one may be varied relative to the other in an arbitrary way; (c) For two weakly coupled superconductors, the phase of one is related to the phase of the other.
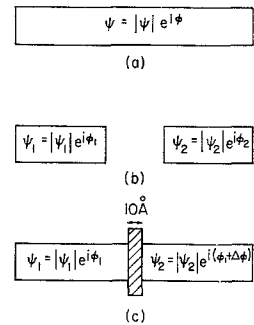
phase factor. Now consider the system consisting of two superconductors separated by an insulating barrier (see Fig. 1). As the barrier thickness is reduced to zero, the properties of the system change continuously from those of two isolated superconductors to those of a single superconductor. The coupling energy of the two sides thus increases (that is, becomes more negative) as we reduce the thickness of the barrier, and once this energy exceeds the thermal fluctuation energy of the system, the two superconductors become phase locked. When this locking is finite but very weak, the effects first described by Josephson may occur. For example, it is now possible for Cooper pairs to tunnel through the barrier from one superconductor to the other and thus to constitute a supercurrent.

In the limit of weak coupling the barrier coupling energy per unit area, E_c, is given by[20,21]

$$E_c(\Delta\phi) = -(\hbar/2e)j_1 \cos\Delta\phi \qquad (2.1)$$

and the supercurrent per unit area by[21,22]

$$j_s(\Delta\phi) = j_1 \sin\Delta\phi. \qquad (2.2)$$

Here j_1 is the maximum supercurrent which the barrier can sustain, and its value depends upon the properties of the barrier. $\Delta\phi = \phi_2 - \phi_1$ is the difference between the phases of the two superconductors and is determined by the current j which is passed through the junction. As j is increased from zero, $\Delta\phi$ increases and the coupling energy, E_c, is weakened. For values of $j < j_1$, the current flows as a direct supercurrent, and no voltage appears across the junction. This is the dc Josephson effect. When $j = j_1$, the coupling energy has been reduced to zero and the junction can no longer sustain a supercurrent. The maximum dc supercurrent which the junction can sustain is
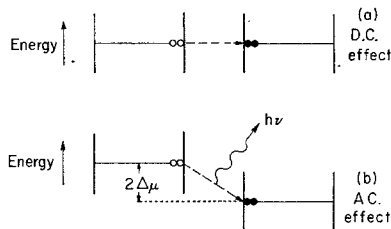
Fig. 2. Schematic representation of dc and ac Josephson tunneling. (a) In the dc effect, pairs are transferred across the barrier between superconductors at the same chemical potential. (b) In the ac effect, there is a chemical potential difference between the two superconductors and the tunneling is accompanied by the emission (in this case) of a photon of energy $h\nu = 2\Delta\mu$.

known as the *critical current*. As j is increased beyond j_1, a finite voltage appears across the junction, and the supercurrent oscillates with time as $j_s = j_1 \sin\omega t$. Part of the current is now carried by single electrons which tunnel through the barrier. In this, the ac Josephson effect, the alternating supercurrents may be regarded as being due to the virtual tunneling of Cooper pairs through the barrier, together with the emission or absorption of a photon[14] (see Fig. 2). Now if there is a finite electrochemical potential difference $\Delta\mu$ between the two superconductors, the Cooper pairs on opposite sides of the barrier differ in energy by an amount $2\Delta\mu = 2eV$. Thus, the photon frequency (ν) will be given by the Josephson voltage–fequency relation

$$h\nu = 2\Delta\mu. \qquad (2.3)$$

A voltage difference of $1\ \mu V$ gives a frequency of roughly 500 MHz. By measuring the chemical potential difference $\Delta\mu$, which corresponds to a frequency ν, we can use Eq. (2.3) to determine $2e/h$. Clearly, the accuracy of Eq. (2.3) is of paramount importance in these measurements, and we shall now investigate the origins of the Josephson equations in some detail.

Let us consider first the two isolated superconductors, 1 and 2, which do not interact with each other.[21,22] We suppose that the superconductors contain N_1 pairs and N_2 pairs, respectively. The ground state Hamiltonian for the system, $\mathcal{K}_0$, has eigenfunctions of the form

$$\Psi_0 = A\psi_0^{(1)}(N_1)\psi_0^{(2)}(N_2) \qquad (2.4)$$

with corresponding energy

$$E_0 = E_0^{(1)}(N_1) + E_0^{(2)}(N_2). \qquad (2.5)$$

$\psi_0^{(1)}$ and $\psi_0^{(2)}$ are just the wave functions used to describe the condensate of superconductors 1 and 2, respectively. Keeping the superconductors isolated, suppose that we now transfer n pairs from 1 to 2 and obtain a new wave function

$$\Psi_n = A\psi_0^{(1)}(N_1-n)\psi_0^{(2)}(N_2+n) \qquad (2.6)$$

and a corresponding energy

$$E_n = E_0^{(1)}(N_1-n) + E_0^{(2)}(N_2+n) + (2e)^2 n^2/2C, \qquad (2.7)$$

where C is the capacitance between the two superconductors. Equation (2.7) may be rewritten as

$$E_n = E_0^{(1)}(N_1) + E_0^{(2)}(N_2) + 2en(\mu_1 - \mu_2)$$
$$+ (2e)^2 n^2/2C, \qquad (2.8)$$

where $E_0^{(1)}(N_1)$ and $E_0^{(2)}(N_2)$ are the energies of the neutral superconductors, and μ_1 and μ_2 the chemical potentials. If we neglect the capacitive term and assume that $\mu_1 = \mu_2$ so that no work is done in transferring a pair from 1 to 2, we seen that $E_n = E_0$ and that the Ψ_n are essentially degenerate. As we bring the superconductors together, there will eventually be phase coherence between them. The energy associated with this coherence resolves the degeneracy and represents the coupling energy of the junction, which we shall calculate shortly.

Let us consider the two superconductors with a thin insulating barrier separating them. We may represent the junction by electron pairs in a potential well in each superconductor with a high potential barrier between each well. The wave function of a pair approaching the barrier in superconductor 1 decays exponentially through the barrier and joins onto a propagating wave function in superconductor 2. This is the textbook example of tunneling through a square barrier in one dimension, except that we have used pairs rather than single electrons. This tunneling process, which has a very low probability of occurring, may be represented by a Hamiltonian, $\mathcal{K}^{(2)}$, which removes a pair from 1 and injects it into 2. We shall write down the details of $\mathcal{K}^{(2)}$ although they are not essential to an understanding of the over-all argument.

The tunneling Hamiltonian $\mathcal{K}_T$ of Cohen, Falicov, and Phillips[23] transfers *single* electrons

across the barrier:

$$\mathcal{H}_T = \sum_{k_1 k_2} T_{k_1 k_2}(C_{k_2\uparrow}^+ C_{k_1\uparrow} + C_{k_1\downarrow}^+ C_{k_2\downarrow})$$

$$+ \text{Hermitian conjugate.} \quad (2.9)$$

The operator[24] $C_{k_2\uparrow}^+$ creates an electron in state k_2 with spin $\uparrow$ in superconductor 2, and $C_{k_1\uparrow}$ destroys one in state k_1 with spin $\uparrow$ in superconductor 1. $T_{k_1 k_2}$ is the tunneling matrix element. This form of the Hamiltonian assumes that electrons are transferred instantaneously from one superconductor to the other and that they do not interact in the barrier. $\mathcal{H}_T$ represents a single-particle interaction, and to obtain pair tunneling and resolve the degeneracy we must go to second order and use the Hamiltonian[25]

$$\mathcal{H}^{(2)} = \mathcal{H}_T (E_0 - \mathcal{H}_0)^{-1} \mathcal{H}_T. \quad (2.10)$$

The total Hamiltonian of the system is thus

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}^{(2)}. \quad (2.11)$$

The perturbation transfers pairs one at a time and connects ψ_n whose values of n differ by unity. Ferrell and Prange[26] pointed out that this problem is analogous to that of a one-dimensional lattice of atoms in the tight-bonding approximation.[27] In the latter case, the ψ_n represent the wave function of the electron localized on the nth atom. The eigenstates of $\mathcal{H}$ are of the form

$$\Psi_\alpha = \sum_n e^{in\alpha} \Psi_n, \quad (2.12)$$

where α plays the role of the wave number k. We shall return to the significance of α presently.

The coupling energy of the junction (assuming the Ψ_α to be appropriately normalized) is

$$\langle \Psi_\alpha | \mathcal{H}^{(2)} | \Psi_\alpha \rangle$$

$$= \sum_n \sum_{n'} \langle e^{in'\alpha} \Psi_{n'} | \mathcal{H}^{(2)} | e^{in\alpha} \Psi_n \rangle. \quad (2.13)$$

Now we are concerned only with states whose values of n differ by unity. This approximation is equivalent to considering overlap between nearest neighbor atoms only in the band theory model. Under these circumstances, Eq. (2.13) reduces to

$$\sum_n [\langle \Psi_{n+1} | \mathcal{H}^{(2)} | \Psi_n \rangle e^{-i\alpha}$$

$$+ \langle \Psi_{n-1} | \mathcal{H}^{(2)} | \Psi_n \rangle e^{i\alpha}] = -2M \cos\alpha, \quad (2.14)$$

where M is the matrix element for transfer of pairs and is assumed to be real.

We must now identify α. Suppose that we change the phase of the pairs in one superconductor relative to the phase in the other by an amount α_0.[28] The Ψ_α will transform to $\Psi_n \exp(in\alpha_0)$, so that from Eq. (2.12) we see that Ψ_α becomes $\Psi_{\alpha+\alpha_0}$. Therefore, α represents the phase difference across the junction which we earlier denoted by $\Delta\phi$. Also, it can be shown by evaluating Eq. (2.14) that $M = j_1\hbar/4e$. We have therefore shown that the condensation energy of the barrier is $-(j_1\hbar/2e) \cos\Delta\phi$, as was given by Eq. (2.1).

Following Anderson,[22] we next consider the derivation of the Josephson current and voltage–frequency relations, Eqs. (2.2) and (2.3). The operators $\tilde{n}$ and $\Delta\tilde{\phi}$, corresponding to the quantities n and $\Delta\phi$ which appear in Eq. (2.12) (with $\Delta\phi = \alpha$), are canonically conjugate[24] and satisfy the commutation relation $[\tilde{n}, \Delta\tilde{\phi}] = -i$. This leads us to make the equivalences

$$\tilde{n} = -i[\partial/\partial(\Delta\tilde{\phi})] \quad (2.15)$$

and

$$\Delta\tilde{\phi} = i(\partial/\partial\tilde{n}). \quad (2.16)$$

The time derivatives of $\tilde{n}$ and $\Delta\tilde{\phi}$ are given by the Heisenberg equations of motion[24]

$$i\hbar(\partial\tilde{n}/\partial t) = [\tilde{\mathcal{H}}, \tilde{n}] = i[\partial\tilde{\mathcal{H}}/\partial(\Delta\tilde{\phi})] \quad (2.17)$$

and

$$i\hbar[\partial(\Delta\tilde{\phi})/\partial t] = [\tilde{\mathcal{H}}, \Delta\tilde{\phi}] = -i(\partial\tilde{\mathcal{H}}/\partial\tilde{n}), \quad (2.18)$$

where the right-hand side of each equation was obtained by allowing the commutators to operate on some state function. The mean values of Eqs. (2.17) and (2.18) give

$$\langle \partial\tilde{n}/\partial t \rangle = \hbar^{-1}\langle \partial\tilde{\mathcal{H}}/\partial(\Delta\tilde{\phi}) \rangle = \hbar^{-1}(\partial E/\partial(\Delta\phi)) \quad (2.19)$$

and

$$\langle \partial(\Delta\tilde{\phi})/\partial t \rangle = \hbar^{-1}\langle \partial\tilde{\mathcal{H}}/\partial\tilde{n} \rangle = \hbar^{-1}(\partial E/\partial n) = (2/\hbar)\Delta\mu, \quad (2.20)$$

where by definition $\partial E/\partial n = 2\Delta\mu$ for the transfer of a Cooper pair. This result is also contained in Eq. (2.8). If we insert Eq. (2.1) in Eq. (2.19) and remember that $j = 2e(\partial n/\partial t)$ for a pair current, we obtain Eq. (2.2). More important for the present purpose is the result Eq. (2.20) which implies that the time derivative of $\Delta\phi$ is a function only of the electrochemical potential difference across the junction, $\Delta\mu$.
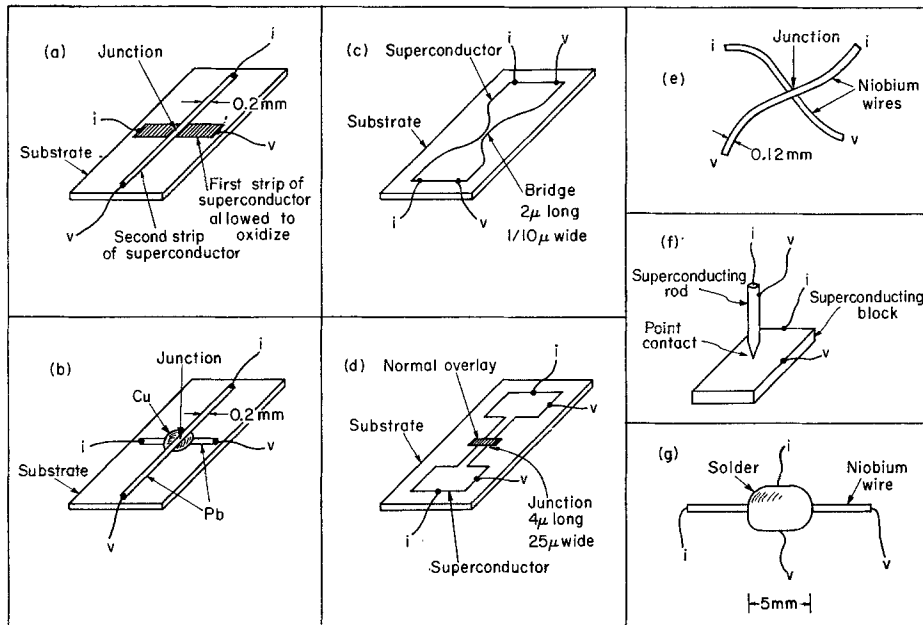
Fig. 3. Types of junction. (a) Thin-film junction with oxide barrier; (b) thin-film junction with metallic barrier; (c) Dayem bridge; (d) Notarys–Mercereau bridge; (e) crossed-wire weak link; (f) point contact junction; (g) slug.

The question of the validity of Eqs. (2.2) and (2.3) now arises. We seen from Eq. (2.19) that the current–phase relation depends on the coupling energy of the junction and is therefore critically dependent upon the form of the tunneling Hamiltonian and the details of the pair tunneling mechanism. For strongly coupled junctions we would expect higher order contributions to the current, of the form $\sin(2\Delta\phi)$, for example. However, relatively weakly coupled junctions appear to be adequately described by the relation $j = j_1 \sin\Delta\phi$.

On the other hand, Eq. (2.20) is completely independent of the form of the tunneling Hamiltonian. The result follows from the time dependence of the coherent wave functions in the superconductors but is not affected by the actual form of the wave function. All of the well known basic properties of superconductors rely implicitly on the existence of phase coherence and there are good grounds for supposing that Eq. (2.20) is an exact relation. We shall return to a further discussion of the validity of the Josephson voltage–frequency relation in Sec. IV.

At this point, we emphasize the importance of the appearance of the electrochemical potential $\Delta\mu$ in Eq. (2.20) rather than of the electrostatic potential eV. In general, $\Delta\mu$ may contain contri-

butions from such sources as the Bernoulli effect and temperature and stress gradients, as well as from the electrostatic potential. However, any "voltmeter" which makes electrical contact with the two sides of the junction in fact measures $\Delta\mu$ rather than eV. Consequently, the quantity measured by a potentiometer (for example), $\Delta\mu$, is the one directly related to the Josephson frequency. Since $\Delta\mu$ may be ultimately related to the standard volt, the Josephson frequency may be expressed in terms of the standard volt without any corrections being necessary (apart from those associated with stray emf's in the potentiometer circuit itself, which are relatively straightforward).

Finally, for completeness, we note that the application of a magnetic field at right angles to the direction of current flow through the junction will modify $\Delta\phi$. If the field may be derived from a vector potential $\mathbf{A}$, the current–phase relation becomes[20]

$$j = j_1 \sin[\Delta\phi - (2e/\hbar c)\int \mathbf{A}\cdot d\mathbf{s}], \quad (2.21)$$

where the integral is taken along a curve joining the superconductors. This result has been confirmed by a number of elegant experiments[29] in which the critical current was measured as a function of magnetic field. However, there is no
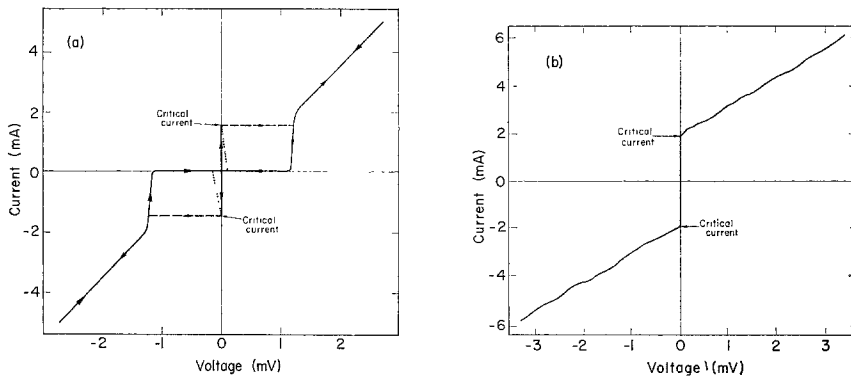
FIG. 4. Current–voltage characteristics. (a) Oxide junctions [Fig. 3(a)], in this case Sn–SnO–Sn at 1.2°K. The continuous line at finite voltage represents the single-particle tunneling curve. For a constant current source the characteristic jumps discontinuously (– – –) from the critical current to the single-particle curve whereas for a voltage source a negative resistance region is observed (...). (b) Weak link, in this case, a slug [Fig. 3(g)].

theoretical basis or experimental evidence for believing that a magnetic field affects the voltage–frequency relation, Eq. (2.3).

II.2. Experimental Josephson Junctions

Figure 3 illustrates some of the numerous types of "weak-link" which have been studied. The dc Josephson effect was first discovered experimentally by Anderson and Rowell[20] using tunnel junctions of the type shown in Fig. 3(a). These junctions are made by first evaporating a strip of tin or lead on to a glass substrate at low pressures. The surface of the strip is then oxidized to produce a 10–20 Å layer of oxide, and lastly a second strip of superconductor is evaporated over the first and at right angles to it. The room-temperature resistance of the junctions so formed, measured in the four-terminal arrangement indicated, is usually in the range 10^{-3}–1Ω.

For the study of its low-temperature properties, the junction is immersed in liquid helium. The temperature of the helium bath may be varied between about 1 and 4.2°K by adjusting the pressure above the liquid to the appropriate value with the aid of a mechanical pump.[30] When the two strips of the junction are superconducting, the current–voltage (i–v) characteristic will appear rather like the one shown in Fig. 4(a). The continuous curve at finite voltages represent the current due to the tunneling of *single* electrons.[31] Notice that the single-particle tunneling current is very small at low voltages (the sharp rise occurs at a voltage corresponding to the sum of the energy

gaps of the two superconductors). Suppose, first of all, that we trace out the i–v characteristic using a current source, that is, a source whose impedance is much greater than the junction impedance. As we increase the current from zero, no voltage appears until the critical current is exceeded, at which point there is a discontinuous voltage jump to the single-particle curve. As we further increase the current, the voltage follows this curve. If we reduce the current to zero, the voltage may follow the single-particle characteristic all the way down to zero current, or it may jump back to zero at some current below the critical value.

On the other hand, if we supply the current from a voltage source whose impedence is much less than that of the junction, we shall observe a somewhat different behavior. As the voltage of the source is increased, the zero-voltage current flowing through the junction increases until the critical current is exceeded. At this point a further increase in the voltage causes the current to *decrease* as the negative-resistance region indicated by the dotted line is traced out. Once the current has been reduced to zero, the single-particle curve will be followed for both increasing and decreasing voltages.

The oxide junction exhibits true tunneling behavior in the sense that there are no propagating electron-pair states in the barrier. The Josephson current is believed to obey the equation $j = j_1 \sin\Delta\phi$ quite accurately at any point.

A related type of junction is one in which the oxide barrier is replaced by a nonsuperconducting

metal such as copper[32,33] [see Fig. 3(b)]. Typical junction resistances are in the range 10^{-6}–$10^{-7}\Omega$. There are now propagating one-electron states in the barrier and a finite pair amplitude induced by the proximity of the superconductors.[32] However, these pairs are not in propagating states unless the two superconductors are phase locked, in which case a dc supercurrent may flow between them. Consequently, the junction is still a tunnel junction in a sense and the current phase relation is believed to be still valid.

Another kind of junction consists of a superconductor of very small cross section between two bulk superconductors.[34] The supercurrent flows through the weak link in much the same way as it flows in a bulk superconductor, and there is no tunneling process. However, these bridges have many properties in common with tunnel junctions: In particular, they can sustain supercurrents of only a few milliamperes and produce a finite voltage at higher currents. A typical i–v characteristic is shown in Fig. 4(b). There is no negative-resistance region and usually no hysteresis at finite voltages. The current–phase relation is probably far from sinusoidal but nevertheless at finite voltages the phase is periodic in time. Anderson[34,35] has explained this behavior in terms of a flux–flow model in which vortices, each containing one quantum of flux, flow across the junction at right angles to the direction of the current. Each vortex produces a phase change of 2π as it crosses the junction and thus gives rise to the periodic behavior.

The first flux–flow junction was that investigated by Anderson and Dayem[34] who evaporated a thin film of tin in the configuration of Fig. 3(c). A related device has been studied by Notarys and Mercereau[36] who evaporated a strip of normal metal over the constriction [Fig. 3(d)]. The normal metal "depresses" the superconductivity[32] in the bridge so that the critical currents may be kept at a low level. An advantage of this structure is that it may be used well below the transition temperature (T_c) of the bulk superconductor, whereas the Dayem bridge can only be used close to T_c where the link becomes sufficiently weak.

A further group of junctions have barriers which are not very well defined and whose properties may vary between the two extremes

mentioned above. Figure 3(e) shows a junction made by pressing together two niobium wires.[37] The natural oxide layer of the wires forms the barrier although tiny metallic bridges may also be present. The point contact junction [Fig. 3(f)] is made by pressing a sharpened niobium wire onto a block of indium.[37] The characteristics of this device may be widely varied by adjusting the pressure on the point from outside the cryostat. Finally Fig. 3(f) shows the "slug," which is made by freezing a bead of solder (a superconductor below about 7°K) around a length of niobium wire.[8]

For the junctions illustrated in Figs. 3(c)–3(d), the current–phase relation Eq. (2.2) is unlikely to be valid as this result depends critically on the mechanism for the transfer of pairs across the weak link. On the other hand, the voltage–frequency relation Eq. (2.3) is very general and not dependent upon the details of the barrier. Consequently, the relations $h\nu = 2\Delta\mu$ should be universally valid, so that, at least in principle, any of these junctions could be used in the e/h measurement.

II.3. Methods of Measuring e/h

There are basically two methods by which we can simultaneously measure the frequency of the ac Josephson supercurrent and the corresponding voltage. In the first, external microwaves of angular frequency ω_0 are applied to the junction. If the ac supercurrent is at a frequency $n\omega_0$, where n is an integer, the two fields may become locked together to produce a current step in the i–v characteristic, as shown in Fig. 5. Shapiro[38] was the first to observe these effects experimentally. The applied microwaves induce an ac
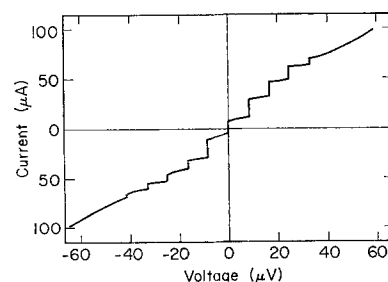

Fig. 5. Constant-voltage current steps induced on Sn–SnO–Sn junction by 4-GHz radiation. The step spacing is about 8.5 μV. The temperature was 3.04°K, and the ambient field 1.1 G.

voltage across the junction which frequency modulates the ac Josephson current in the junction, producing a whole series of sidebands. The constant-voltage current steps are just the zero frequency or dc sidebands. If we assume that both the dc current and the microwaves have low-impedance sources, we can quite simply calculate the form of the induced steps.[14] The total voltage across the junction is

$$V = V_0 + V_1 \cos\omega_0 t, \qquad (2.22)$$

where V_0 is the dc voltage and V_1 the amplitude of the ac voltage induced by the microwave field. Inserting Eq. (2.4) into Eq. (2.3), setting $\Delta\mu = eV$, and integrating, we find

$$\Delta\phi(t) = (2eV_0/\hbar)t + (2eV_1/\hbar\omega_0)\sin\omega_0 t + \alpha, \qquad (2.23)$$

where α is a constant. Thus from Eq. (2.2),

$$j(t) = j_1 \sin\left(\frac{2eV_0}{\hbar}t + \frac{2eV_1}{\hbar\omega_0}\sin\omega_0 t + \alpha\right)$$

$$= j_1 \sum_{n=-\infty}^{\infty}\left\{J_n\left(\frac{2eV_1}{\hbar\omega}\right)\sin\left[\left(n\omega_0 + \frac{2eV_0}{\hbar}\right)t + \alpha\right]\right\}, \qquad (2.24)$$

where J_n is the Bessel function of order n. In order to observe dc effects, the time average of the supercurrent $j(t)$ must be nonzero. When $n\hbar\omega_0 = -2eV_0$, $j(t)$ has a nonzero mean value:

$$j_{dc} = (-1)^n j_1 J_n(2eV_0/\hbar\omega_0)\sin\alpha. \qquad (2.25)$$

This result predicts a series of *spikes* in the direct current at voltages $n\hbar\omega_0/2e$ ($n = 0, \pm1, \pm2, \cdots$).
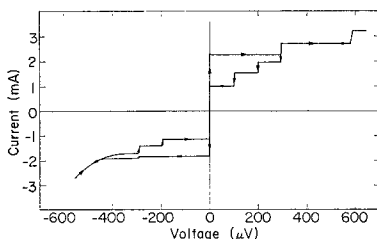

FIG. 6. Self-induced steps on Sn–SnO–Sn junction at 2.08°K. The width of the junction in which the standing waves were set up was about 0.16 mm, and velocity in the barrier corresponding to the voltage spacing was about 2×10^9 cm sec^{-1}. A field of 2 G was applied (in the plane of the junction and at right angles to the direction of the standing wave) to maximize the step amplitude. Note the hysteresis and that some of the steps are nonvertical.

If, as is more usual in practice, the dc current has a high source impedance, current *steps* rather than spikes will be induced with no negative-resistance regions (see Fig. 5). We see from Eq. (2.25) that j_{dc} may be either in the direction of the dc current or against it, and power is accordingly absorbed from or supplied to the microwave field. The size of the step is an oscillatory function of microwave amplitude.

In the special case of an oxide junction subharmonic effects at voltages $n\hbar\omega_0/(m2e)$ are not normally observed. This result follows from the simple $\sin\Delta\phi$ dependence used in (2.24). However, in most other kinds of weak link in which the current is a more general function of $\Delta\phi$, subharmonic steps are also observed. These additional steps do not interfere in any way with the *voltage* at which the integer steps appear. A detailed description of the structure and origin of induced steps has been given by Clarke, Pippard, and Waldram.[39]

The appearance of the steps has a simple quantum interpretation[14] based on the process of virtual pair tunneling with emission and absorption of photons described in Sec. II.1. The pairs can absorb (or supply) photons from (or to) the microwave field whenever the energy difference across the barrier corresponds to the photon energy $\hbar\omega_0$ or a multiple of it. Subharmonic effects occur when the absorption or emission of n photons by m pairs is allowed.

It is evident that the microwave-induced steps provide a very simple technique (in principle) for measuring $2e/h$. The microwave frequency may be readily determined to one part in 10^8, and one needs only to measure the voltage at which a step of known order appears.

The second method of measuring $2e/h$ relies on the self-induced step structure first observed by Fiske[40] in tunnel junctions (see Fig. 6). If the frequency of the ac supercurrent corresponds to a resonant frequency of the junction, the supercurrent couples strongly to the electromagnetic fields of the mode to produce a small voltage across the junction[41] and set up a standing wave. If we regard the junction as a parallel plate transmission line, the voltages at which steps appear are given by

$$V_k = \hbar\omega_k/2e = k(\hbar\pi\bar{c}/2eL). \qquad (2.26)$$

k is an integer, L the length of the junction, and $\bar{c}$ the propagation velocity of the electromagnetic wave.[42] When the junction is biased on a self-induced step, it emits radiation at the corresponding frequency and also at multiple and subharmonic frequencies. This method was used for the first direct observation of the ac effect by Yanson et al.[43] Similar effects have been observed by placing a point contact in a resonant cavity.[10] Although in principle we could use this self-resonant technique to measure e/h by detecting the radiation and measuring its frequency, there are several drawbacks. First, the amount of power radiated is small, on the order of picowatts. Second, these effects can be observed only up to frequencies of perhaps 10 GHz with a corresponding voltage of $20\,\mu V$, a voltage too small to be measured to 1 ppm. Third, the steps are not truly vertical, being limited in this respect by the Q of the cavity, whereas the microwave-induced steps are relatively sharp (but see Sec. IV). Finally, Langenberg et al.[44] have discovered that when higher-order self-induced steps are used, radiation may be emitted at a frequency that does not correspond to the correct voltage $n\hbar\omega/2e$. Consequently, this second technique is of limited applicability, and the high-accuracy measurements have made use of the microwave-induced steps.

III. EXPERIMENTS TO MEASURE e/h

In this section we shall be most concerned with the measurements performed by Parker, Langenberg, Denenstein, and Taylor.[11] The interested reader should consult their paper[11] to gain some idea of the extraordinary precautions necessary to perform measurements to a precision of 1 ppm. At the same time he should bear in mind that this experiment is one of the simplest available to determine a fundamental constant.

The experiments were performed upon both tunnel and point contact junctions made of a variety of different superconductors. For the work with microwave-induced steps two frequencies were available, 10 GHz and 70 GHz. A few measurements were taken by measuring the frequency of the radiation emitted from self-induced steps, but these measurements were of a much lower accuracy and do not contribute significantly to the final results. The two methods

gave the same result to within the experimental error. We shall describe, as an example, the measurements made on induced steps in tunnel junctions at 10 GHz although the general technique was common to point contacts and to the higher frequency.

III.1. Cryogenic and Microwave Equipment

A sketch of the cryostat appears in Fig. 7. The outer Dewar contains liquid nitrogen, and the inner, liquid helium, whose temperature may be lowered to about 1.2°K by pumping on the vapor above it. The temperature of the bath was estimated from the vapor pressure of the helium.[30] A mu-metal shield around the cryostat reduced the earth's field to a low level and minimized the pickup of external rf. A Helmholtz pair of coils in the nitrogen bath was available to supply magnetic fields of a few gauss to the junctions. The microwaves were fed down a waveguide to the junction inserted at the lower end. The position of the short circuit plunger below the junction could be adjusted from the top of the cryostat to vary the coupling between junction and waveguide. Various orientations of the junction to the waveguide and to the applied field were tried. The copper leads from the junction were brought through an insulated feedthrough at the top of the cryostat and connected directly to the terminals of the potentiometer in order to minimize thermal emfs.
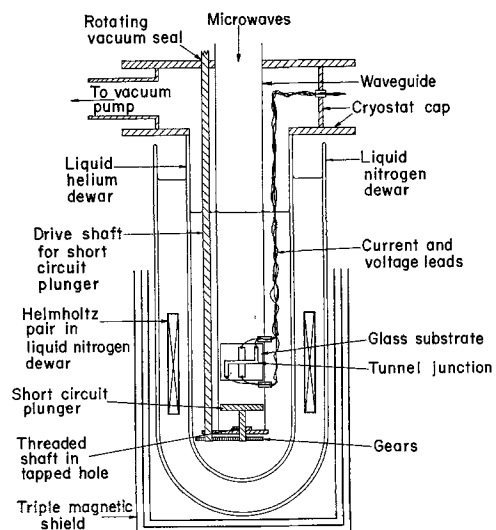

Fig. 7. Cross section through a typical cryostat for measuring $2e/h$ (based on a drawing of Parker et al.[11]).

The microwave equipment was fairly conventional. The microwave oscillator was stabilized by phase locking it to a quartz-crystal reference source which had a stability of a few parts in 10^8 over a period of an hour. The oscillator frequency was measured by a counter which was calibrated against the US frequency standard broadcast by WWVB in Fort Collins, Colorado. These techniques yielded a frequency measurement better than one part in 10^8, an error two order of magnitude less than the other errors in the experiment. The power fed to the junction was estimated by a bolometer.

III.2. Voltage Measurement

The most difficult part of the whole experiment was the determination of the absolute voltage at which a given step appeared. However, before describing the measurements on the tunnel junctions, we shall make some pertinent remarks on the status of the absolute volt.

Banks of cells provide the basic reference standard for the unit of electromotive force in the various standards laboratories of the world. These cells are calibrated in terms of the absolute volt determined by passing a current of one absolute ampere through a resistance of one absolute ohm. The uncertainty in the absolute volt is about 6 ppm. In addition to the uncertainty of the absolute voltage of the standard cells at the time of calibration, further errors may arise from the

Fig. 8. Simplified diagram of voltage measuring circuit (based on diagram of Parker et al.[11]).

tendency of the cells to change with time, by as much as 1 ppm/year. The value of $2e/h$ obtained by Parker et al. was expressed in terms of the average voltage of a bank of standard cells over a period of about 1 yr. There are, therefore, two types of uncertainty in the absolute voltage measurement. One arises from the errors associated with the experiment itself and from the calibration of the cells in terms of the NBS standard cells. The second uncertainty is in the knowledge of the NBS volt in terms of the absolute volt.

We return now to the determination of $2e/h$. The largest voltages at which steps could be induced on the junctions were on the order of 1 mV. A knowledge of these voltages in terms of the as-maintained NBS volt was required to an accuracy of 1 ppm, and it was therefore necessary to measure 1 mV to at least six significant figures.

The essential components of the voltage-measuring circuit are illustrated in Fig. 8. A constant current I is fed into the slider of a potentiometer P across which is connected a chain of dividing resistors. These resistors were immersed in oil in a sealed box in order to stabilize the temperature and minimize self-heating effects. I is chosen so that when the potentiometer slide is at A, a voltage of precisely 1 V appears at A and thus across the chain of resistors. The latter have been chosen to provide voltages of 0.001 V, 0.01 V, and 0.1 V at the taps. Provided I remains constant, the voltage appearing across the resistor chain is linearly proportional to the potentiometer setting. We thus have a potentiometer with four decade ranges within each of which the accuracy of the voltage depends on the linearity and resolution of the potentiometer P. The potentiometer was a seven-decade Kelvin–Varley divider capable of yielding seven significant figures with a maximum error of 0.3 ppm. An additional series resistor S whose value was chosen to give a potential drop of 0.1 V produced a further terminal at -0.1 V. Thus, the -0.1 V and 1 V terminals had a range of 1.1 V and could be used to calibrate the potentiometer against a standard cell. In fact, six standard cells, calibrated by the NBS in terms of the as-maintained US legal volt, were used. They were kept in a constant-temperature air bath to minimize drifts due to temperature fluctuations.

The actual measurement of the voltage across the junctions made use of a conventional null
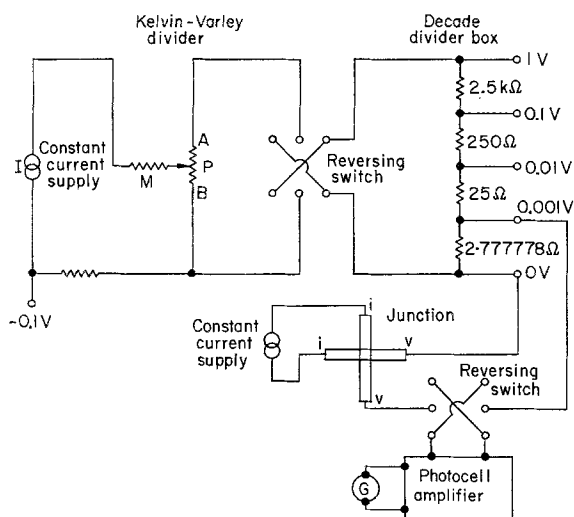
TABLE I. Values obtained for $2e/h$ from Josephson junctions.

Parker et al[11]:

Weighted average of 19 runs for induced steps in terms of local volt	$483.5978(\pm 0.4$ ppm) MHz/μV_L
Weighted average of two runs for radiation emission in terms of local volt	$483.5985(\pm 3.4$ ppm) MHz/μV_L
Weighted average for all data in terms of local volt	$483.5978(\pm 0.4$ ppm) MHz/μV_L
Final value after conversion to NBS68 volt	$483.5976(\pm 2.4$ ppm) MHz/μV_{NBS68}
Final value after conversion to NBS69 volt	$483.5935(\pm 2.4$ ppm) MHz/μV_{NBS69}
Final value in terms of absolute volt (see text)	$483.5933(\pm 3.6$ ppm) MHz/μV_{abs}

Petley and Morris[45]:

Final value for induced steps in terms of NPL69 volt	$483.5939(\pm 2.2$ ppm) MHz/μV_{NPL69}
Final value for induced steps after conversion to NBS68 volt (see text)	$483.5977(\pm 2.2$ ppm) MHz/μV_{NBS68}

technique with a low-thermal switch and a photocell amplifier having a resolution of about $\frac{3}{4}$ nV. The procedure was as follows. The i–v characteristic of the junction was observed on an oscilloscope, and the amplitude of the steps was adjusted by varying the amplitude and frequency of the microwaves, the position of the short-circuit plunger, and the value of the magnetic field. The microwave source was stabilized and its frequency measured. A dc current was then passed through the junction to bias it on the nth order induced step and the voltage so developed measured with the potentiometer. The junction current was reversed, it was adjusted if necessary to keep the junction on the step, and the voltage was remeasured. The voltage at which the nth step appeared was then taken as the average of the two readings. This averaging procedure eliminated thermoelectric voltages in the cryostat leads. Sometimes different order steps for each polarity were used, and subharmonic steps were also measured. All measurements gave the same value of $2e/h$. Spurious effects due to Peltier emfs, which reverse with current, and to the rectification of the microwaves or external pickup were shown to be negligible.

On some of the induced steps, it was found that the voltage was slightly dependent upon the current, that is, that the steps were not truly vertical. Under these circumstances, the voltage at the midpoint of the step was taken to be the correct one. We shall discuss this effect further in Sec. IV.

III.3. Numerical Results

The data from 19 runs on radiation-induced steps gave a mean value of $2e/h = 483.5978$ (± 0.4 ppm) MHz/μV_L (see Table I). The uncertainty is the random error (one standard deviation) of the readings and the result is expressed in terms of the local laboratory volt. The two runs in which radiation emission data were obtained yielded a mean value of 483.5985 (± 3.4 ppm) MHz/μV_L, which is well within one standard deviation of the value obtained from the induced-step experiments. The random error of these measurements was large because of the relatively low voltage at which they were taken (see Sec. II.3), and they did not in fact contribute significantly to the final weighted average value.

The mean value of $2e/h$ was then expressed in terms of the NBS standard volt as maintained in 1968. This conversion involved a number of possible systematic errors arising, for example, from the uncertainty in the NBS calibration of the standard cells, the uncertainty in the standardization of the potentiometer, and errors due to temperature drifts in the decade divider. The value obtained in terms of the 1968 NBS volt was 483.5976 (± 2.4 ppm) MHz/μV_{NBS68}. However, on 1 January 1969, by international agreement, the various national laboratories adjusted their as-maintained voltage standards in order to bring them closer to the absolute volt. The resultant change in the NBS volt produced a value for $2e/h$ of 483.5935 (± 2.4 ppm) MHz/μV_{NBS69}.

It remains to convert the NBS volt to the absolute volt. Unfortunately, as we have already mentioned, the conversion factor is known to only 6 ppm. Taylor, Parker, and Langenberg have consequently performed a new least-squares adjustment of the fundamental constants in which the ratio of the NBS and absolute amperes are regarded as an adjustable parameter to obtain a more precise value of V_{NBS}/V_{abs}. We shall

briefly outline this procedure in Sec. VI.3. Their value for $2e/h$ in terms of the absolute volt is 483.5933 (± 3.6 ppm) MHz/μV$_\text{abs}$. This value is 38 ± 10 ppm smaller than that given by the Cohen and DuMond[12] 1963 least-square adjustment of the fundamental constants. We shall discuss the significance of this discrepancy in Sec. VI.

III.4. An Independent Determination of $2e/h$

Petley and Morris[45] have repeated the $2e/h$ measurement at the National Physical Laboratory (NPL) in the United Kingdom. Their experiments were on the slug type of junction [Fig. 3(f)] made of a variety of different materials, and they used the induced-step technique at a frequency of 36.8 GHz. Their value of $2e/h$ was 483.5939 (± 2.2 ppm) MHz/μV$_\text{NPL69}$ where V$_\text{NPL69}$ refers to the value of the standard volt maintained by the NPL after 1 January 1969.

Since there has been no official-standards comparison following the 1969 adjustment, a direct comparison with the value found by Parker et al. is a little difficult. Perhaps the simplest way is to convert the 1969 NPL volt back to its pre-1969 value and then express it in terms of the 1968 NBS volt which, if we assume the 1967 standards comparison to be valid, was 5.2 ppm lower than the pre-1969 NPL volt. On these assumptions, the value for $2e/h$ found by Petley and Morris becomes 483.5977 (± 2.2 ppm) MHz/μV$_\text{NBS68}$. This value is 0.2 ± 3.3 ppm higher than that of Parker et al. At face value, this agreement is, of course, remarkably good, the error being much less than one standard deviation. However, it seems likely that further minor adjustments are inevitable.[46] In the first place, the conversion of the NPL 69 volt to the NBS 68 volt which we described is in fact unlikely to be accurate to better than perhaps 1 ppm, judging by previous drifts in the as-maintained voltage standards. Secondly, it seems that both sets of authors are considering final adjustments to their results on the order of a few $\frac{1}{10}$ ppm.[46] Nevertheless, it seems very probable that the final results, expressed in terms of a common volt by means of a direct NBS–NPL comparison, will agree to within one standard deviation. The fact that two independent determinations in different countries are in such good agreement is most encouraging.

IV. THE ACCURACY OF THE JOSEPHSON VOLTAGE–FREQUENCY RELATION

How precise is the Josephson voltage–frequency relation, $\hbar\omega = 2\Delta\mu$? We saw in Sec. II that there were good theoretical grounds for believing it to be exact. In this section, we shall be concerned with the experimental evidence which demonstrates the precision of the voltage–frequency relation and indicates that it is a fundamental constant independent of the conditions under which the experiment is performed. We shall also discuss some relevant theoretical ideas which have recently been put forward.

IV.1. Type of Junction

Parker et al.[11] used both oxide tunnel junctions and point contacts in their experiments. These two types of junctions presumably had different microscopic mechanisms for the pair transport process and certainly quite different geometries. There was no difference in the values of $2e/h$ obtained to within 0.6 ppm.

IV.2. Junction Material and Experimental Parameters

Parker et al.[11] examined junctions made of five different superconductors which differed in such parameters as the electron–phonon interaction, Fermi energy, coherence length, and so on. Variation of the temperature of the junctions over the range $0.3 \leq T/T_c \leq 0.9$ gave rise to changes in the energy gap, density of condensed pairs, coherence length, and penetration depth. The data were taken at two different frequencies (10 GHz and 70 GHz), and the microwave power could be changed by at least an order of magnitude. The Helmholtz coils (Fig. 7) were used to apply magnetic fields to the junctions of up to 10 G. The tunnel junctions were placed in the waveguide in different orientations so that the coupling was predominantly by means of either the electric field or the magnetic field. The value of $2e/h$ measured with the different junctions and under the different experimental conditions remained constant to within 2 ppm.

The present author[47] has performed a comparison experiment in which he demonstrated that the voltage–frequency relation was the same in lead, tin, and indium to a precision of one part in 10^8. The experiments were performed on Pb–Cu–Pb,
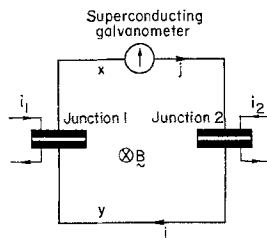
FIG. 9. Circuit for comparing the Josephson voltage–frequency relation in junctions made of different materials and for measuring the dynamic resistance of induced steps in Pb–Cu–Pb junctions.

Sn–Cu–Sn, and In–Cu–In junctions [see Fig. 3(b)]. These very low-resistance junctions were chosen because the extremely small dynamic resistance of the induced steps (see Sec. IV.5) made very precise voltage comparisons possible. The basic principle of the experiment is indicated in Fig. 9. Two junctions made of different superconductors, say lead and indium, were connected in series in a superconducting circuit containing a slug [Fig. 3(g)] used as a superconducting galvanometer.[8] The galvanometer, which had zero resistance, could detect a circulating current as low as 0.3 μA. Radiation at about 1 MHz was used to induce steps on the junctions and the currents i_1 and i_2 were adjusted so as to bias each junction on the midpoint of, say, the first order induced step. Now the difference in chemical potential along each superconducting lead (including the galvanometer) was clearly zero. Let us suppose that the same radiation could induce steps at different chemical potentials $\Delta\mu_1$ and $\Delta\mu_2$ on junctions 1 and 2, respectively. Then there would be a net chemical potential difference $\Delta\mu_1 - \Delta\mu_2$ around the circuit. This would thus induce a circulating current j at the rate

$$dj/dt = (\Delta\mu_1 - \Delta\mu_2)/eL, \qquad (4.1)$$

where L is the circuit inductance. At the beginning of the experiment there was no current flowing around the circuit ($j=0$). After 30 min, this current remained zero to within the resolution of the galvanometer (Δj). Taking the values $\Delta j = 0.3$ μA and $L = 10^{-7}$ H, we find from Eq. (4.1) that $|\Delta\mu_1 - \Delta\mu_2|/e < 1.7 \times 10^{-17}$ V. Since the voltage of each induced step at 1 MHz was about 2 nV, we can say that $|\Delta\mu_1 - \Delta\mu_2|/\mu_1 < 10^{-8}$. The same result was obtained when the pairs of superconductors were lead and indium, or tin and indium, and also when silver was substituted for copper as the barrier in one of the junctions. In addition, the variation of the following parameters did not produce any observable difference in the chemical

potentials across the two junctions: temperature, from 1.2 to 2.2 K; barrier thickness; the level of applied rf power over a factor of 5; the rf frequency, from 100 kHz to 1 MHz (to one part in 10^7 at 100 kHz); the position on the induced step; the ambient magnetic field up to ± 1 G; the direction of the bias current through the junctions.

It is apparent that the chemical potential induced on the junctions at a given frequency is independent of the junction material and the experimental conditions to one part in 10^8. This result is a useful one because if there are no *differences* in the corrections between different materials at the level of one part in 10^8, there is a very good chance that there are no significant corrections due to material-dependent effects at the level of 1 ppm.[48]

IV.3. The Order of Induced Step

In their original measurements, Parker et al.[11] showed that to within 1 ppm the value of $2e/h$ was independent of the order (n) of the step upon which the measurement was made for values of n between 20 and 70. Precise comparisons for lower values of n were not possible because the voltage involved was too small to be measured to 1 ppm.

Finnegan, Denenstein, Langenberg, McMenamin, Novoseller, and Cheng[49] have subsequently compared the value of $2e/h$ obtained from a high-order step induced at about 10 GHz with that obtained from the first-order step induced by a HCN laser at about 900 GHz. The experimental arrangement is indicated in Fig. 10. Two separate junctions were used because it was not possible to induce the higher-frequency steps in the thin-film junction, and the lower-frequency steps induced in the point contact were of very small amplitude. The point contact was adjusted to give $n = 1$ steps induced by the laser at a current below the critical current of the thin-film junction, and the voltage on the step measured with the potentiometer described in Sec. III.3. The voltage was about 1.84 mV. The point contact was readjusted to have a higher critical current so that measurements could be made on the tunnel junction. The microwave frequency and the step number were adjusted so that the voltage was again about 1.84 mV ($n = 94$). The two values of $2e/h$ obtained differed by 0.4 ± 1.5 ppm. Since this result was obtained from two junctions of different types

and different materials, the agreement could conceivably be due to the cancellation of a frequency-dependent effect by a material or junction-type effect. However, the preceding two sections shows that these two possibilities are in fact eliminated. Thus, we may use the present experiment to extend the possible range of frequency over which $2e/h$ may be measured by an order of magnitude and in particular, to remove the possibility of a discrepancy between the very low-order steps and the higher-order steps.

IV.4. High-Order Tunneling Processes

Tunneling processes involving the coherent transfer of, for example, four electrons rather than two may possibly occur, leading to a voltage–frequency relation $n\hbar\omega = 4eV$. If n is even, the step of order n induced by microwaves for the four-electron process would coincide with the step of order $n/2$ for the pair process. If n is odd, steps would appear at voltages corresponding to half-integral steps for the pair process. However, subharmonics may be present in any case both for strongly coupled tunnel junctions (see Sec. II.1) and for point contacts in which flux–flow occurs. Consequently, the higher-order process, which has a very low probability of occurring, is almost impossible to identify and does not affect the accuracy of the $2e/h$ measurement.

One may also raise the possibility of a process in which on the average $2+\delta$ electrons are transferred rather than exactly 2, where δ is a very small quantity. There is no theoretical or experimental evidence for such a conjecture. The experiments described in Sec. IV.3 suggest that δ does not depend upon the properties of the superconductor or the experimental conditions at the level of one part in 10^8. It therefore seems very unlikely that δ is as large as 10^{-6} if indeed it is finite at all.

IV.5. Effects of Noise

The simple theory presented in Sec. II.3 suggests that the current steps induced on the i–v characteristic by external monochromatic radiation should be truly resistanceless. Furthermore, the radiation emitted from a junction biased on a self-induced step should be monochromatic at least to within the stability of the bias voltage. (However, since the slope of the self-induced
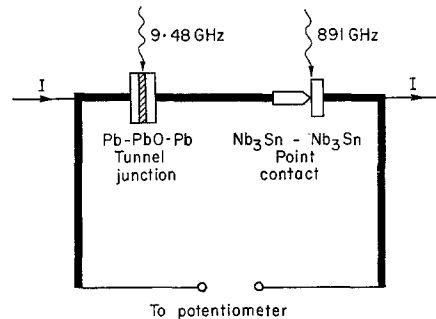

FIG. 10. Diagram showing comparison of voltage of $n=1$ step induced by laser with that of the $n=94$ step induced at microwave frequencies. The heavy line indicates superconducting connections (based on diagram of Finnegan et al.[49]).

current step is usually nonvertical due to the finite Q of the junction cavity, one would expect the frequency to depend upon the actual position on the step.) In practice, as noted in Sec. III.2, the radiation-induced steps are sometimes noticeably nonvertical, and in addition, the radiation emitted from junctions has a finite bandwidth.[50,51] These effects are the result of noise, both noise generated in the junction itself and that picked up from external sources. Our present concern is to examine its effect upon the accuracy of the $2e/h$ determination.

We consider first the finite linewidth of the emitted radiation. We assume that the Josephson current can be written in the form[50,51]

$$j = j_1 \sin\{\omega_0 t + (2e/\hbar)\int V_n(\tau)\,d\tau\}, \qquad (4.2)$$

where V_n is a noise voltage produced across the junction and $\omega_0 = 2eV_0/\hbar$. V_n thus produces random fluctuations in the phase of the Josephson current. There are two contributions to V_n from the current flowing through the junction. The first was calculated by Scalapino[50] and arises from fluctuations in the normal current. The noise is just the shot noise of the single electrons tunneling through the barrier. The second term is due to the shot noise of the pairs and was calculated by Stephen.[52] He regarded the junction in essentially the same way as a laser cavity. Each time a photon decays in the cavity, a pair tunnels to replace it so that, if we assume that the photon dissipation is a random process, the resultant pair tunneling current will also be random and will contain shot noise. The origin of the pair contribution to the

linewidth in the Josephson effect is thus very similar to the origin of the linewidth of a laser. However, when the pairs tunnel via interaction with the photons, the charge flowing across the capacitance produces voltage fluctuations about the mean value. This effect gives rise to a frequency modulation of the radiation and explains why the linewidth of the Josehpson radiation, at best about one part in 10^7, is so much broader than that of the laser radiation, which is perhaps one part in 10^{15}.

The total linewidth in the limit $eV_0 \ll kT$ is given by[52]

$$\Delta \nu = 4e^2 R_D{}^2 k T I_T / \pi \hbar^2 V_0, \qquad (4.3)$$

where R_D is the dynamic resistance of the step, $(di/dV)^{-1}$, and I_T is the total current, that is, the sum of the quasiparticle and pair currents. The line shape is approximately Lorentzian.

Dahm, Denenstein, Langenberg, Parker, Rogovin, and Scalapino[53] have compared their experiments with Eq. (4.3) and found satisfactory agreement. Typical linewidths at a central frequency of 10^{10} Hz vary from less than 1 kHz at low values of temperature and dynamic resistance to 10 MHz near the critical temperature of the superconductors. It appears that the linewidth of the radiation is now well understood and that no significant error is introduced if the central frequency is used to determine $2e/h$.

In the absence of an applied signal, the phase of the Josephson junction fluctuates in the random manner described by Eq. (4.2). The phase difference $\Delta \phi$ thus contains a small noise component $\Delta \phi_N$, which may be regarded as performing a random walk process so that its mean square value is nonzero. It is this fluctuation which produces the linewidth broadening. These phase fluctuations are analogous to the Brownian motion of a free particle. If we now apply monochromatic radiation to the junction to induce current steps, a different situation arises. Provided that the signal amplitude is larger than that of the noise fluctuations, the Josephson oscillator will lock onto it so that the phase of the junction is determined by the phase of the applied radiation. Under these circumstances the effect of the fluctuations is very much smaller. The reason for this can be qualitatively understood in the following way: Suppose a small fluctuation changes the phase of the junction

by a small amount $\Delta \phi_N$. Then the applied radiation will tend to pull the junction phase difference back to its previous value. Only if the phase fluctuations are sufficiently great to allow the junction to "slip" by a complete period will the locking-on mechanism be broken and a small resistance be introduced onto the step. The phase fluctuations are now analogous to a Brownian particle in a tilted periodic potential, where the amplitude of the potential is proportional to the strength of the applied radiation. The particle tends to be trapped in one of the potential minima, and a relatively large fluctuation is required to enable it to pass over the potential barrier into the next minimum. This problem has been discussed at some length by Stephen[54] and by Clarke, Pippard, and Waldram.[39]

These ideas may be used to calculate the slope of the induced steps in the presence of noise. Under the influence of noise the step has the form indicated in Fig. 11. The dynamic resistance in the center of the step is extremely low, but it increases roughly exponentially towards the extremities of the step. The distortion of the step due to noise is symmetric about the midpoint so that the use of the midpoint voltage in the e/h measurement seems to be well justified. The agreement of theory with experiment is not well established at the present time. It appears that in most experiments on thin-film and point-contact junctions the noise picked up from external sources, for example local radio stations, dominated that generated in the junctions. Finnegan et al.[49] found that by placing their entire cryostat in a shielded room, the steps induced on the characteristics of both kinds of junction were vertical to within their experimental resolution, 1 nV.

The present author performed a simple experiment on Pb–Cu–Pb junctions [Fig. 3(b)] which
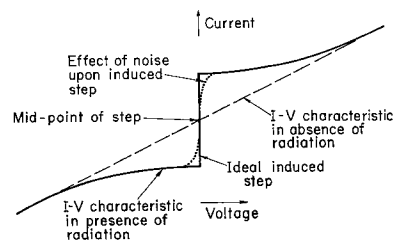

Fig. 11. Sketch of microwave-induced step on current-voltage characteristic of weak link. The dotted line indicates the rounding due to noise.

suggested that the induced steps may in fact be of extremely low differential resistance.[47] At the time of the experiment it was known that the noise effects increased with junction resistance so that a junction in which the resistance was less than 10^{-6} Ω would be expected to have very low inherent noise. In addition, the very low resistance implied that noise picked up from extraneous sources should be small, simply because of the enormous imped-ance mismatch between the junction and the external circuity. The basic circuit used was again that of Fig. 9. Radiation at 1 MHz induced steps on to the i–v characteristics of the junctions, each of which was biased on the first order, say, in-duced step. Thus there was a voltage of roughly 2 nV between X and Y, but the total *dynamic* resistance of the circuit was simply the sum of the dynamic resistances of the two induced steps. The circuit was, in a sense, "differentially supercon-ducting." Thus a magnetic field (B) applied at right angles to the circuit induced a circulating supercurrent j, just as it would for a bulk super-conducting ring. However, this was true only if the supercurrent was sufficiently small that each junction remained biased on its induced step.

An estimate of the differential resistance of the steps was obtained by watching the decay of the induced current by means of the galvanometer. In a typical experiment, the inductance (L) of the circuit was about 10^{-7} H, the circulating current (j) about 0.5 mA, and the decay of the current (Δj) was less than the galvanometer resolution, 0.3 μA, over a period (T) of 30 min. The time constant (τ) of the circuit was given by $\tau \simeq Tj/\Delta j$ and was thus greater than 3×10^6 sec. The total circuit resistance (L/τ) was less than 3×10^{-14} Ω, suggesting that the differential resistance of each induced step was not more than about 10^{-14} Ω. This corresponds to a difference of less than 10^{-17} V between the top and bottom of the step. This value of resistance, of course, applies only to the central part of the step—its value could have been greater towards the extremities. Stephen[54] has estimated that for this kind of junction, the voltage difference would be 10^{-17} V at about 97% of the step height.

The SNS junctions have demonstrated that in principle the induced steps may be exceedingly sharp, but they are unlikely to be of any great use in the $2e/h$ measurements because steps may be induced only at low voltages. However, there seems to be no great difficulty in shielding the experiments on thin-film and point-contact junc-tions so that the steps on all types of junction are quite steep enough for all practical purposes.

IV.6. Frequency-Pulling Effects

Scully and Lee[55] have proposed that the fre-quency of radiation emitted from a resonant junction biased at a voltage V_e may be slightly pulled from the Josephson value of $2eV_e/h$. A similar shift was noted by Stephen.[52] The cal-culation is based on an analogy with a laser cavity. Consider an atom which emits radiation at a frequency ω and which is placed for a finite time in a cavity whose resonant frequency differs from ω. The radiation emitted by the atom in the cavity will be slightly shifted from ω by cavity detuning. Scully and Lee[55] suggest that because of the finite lifetime of the pairs in the superconductor forming the junction, there will be a similar shift in the frequency of the radiation emitted by the junction. They estimate the size of the effect to be on the order of one part in 10^8 for a typical junction.

McCumber[56] has emphasized that frequency pulling will *not* affect the Josephson voltage–frequency relation $2\Delta\mu = \hbar\omega$. As we remarked in Sec. II.1, there may be a number of contributions to the electrochemical potential $\Delta\mu$. The potenti-ometer used in the e/h determination detects $\Delta\mu$ and hence, provides a measure of the *total* energy required to transfer a pair from one side of the junction to the other. This total energy may include a contribution from the interaction of the pair with the electromagnetic fields in the cavity which is responsible for the frequency pulling. Thus the voltage–frequency relation is unaffected by frequency pulling. In any case, the frequency-pulling calculation is applicable only to emitted radiation and not to the induced steps used for the high-precision measurements.

Nevertheless, the presence of a frequency-pulling effect in a junction is an interesting pos-sibility. The effect may possibly be observed by measuring the *electrostatic* potential difference across a radiating junction.

IV.7. Discussion

It appears that there is good experimental evidence for the invariance of the Josephson

voltage–frequency relation under all experimental conditions and thus for believing that the measured quantity is indeed a fundamental constant. In addition, there are at present no theoretical calculations which suggest that a correction should be applied, at least at the level of 1 ppm, although one should not, of course, dismiss the possibility of such a correction being discovered at some future time. For the present, we are forced to assert that the frequency–voltage ratio is exactly $2e/h$. In Secs. V and VI, we shall discuss the implications of this assertion in relation to other areas of physics.

V. THE ac JOSEPHSON EFFECT AND STANDARDS OF ELECTROMOTIVE FORCE

The various national laboratories of the world maintain the standard volt by means of standard cells. We have already seen in Sec. III that this method raises difficulties because of the tendency of the voltage of the cells to drift with time. Another problem is in the comparison of the as-maintained voltage standards of different countries. Every 3 yr or so, various countries check their respective standard cells with those maintained by the Bureau International des Poids et Mesures (BIPM) in France. These checks often reveal drifts of several parts per million, but it is not clear if these drifts arise from the BIPM cells, from the cells of the countries who participate, or from both. It is clear that a method for checking the stability of a standard cell over long periods of time to an accuracy of 1 ppm or better is needed. If different countries could compare their standard cells with the same reference without having to transport them to a distant laboratory, a considerable reduction in the discrepancy between the national standards would result.

Taylor, Parker, Langenberg, and Denenstein[57] have pointed out that the ac Josephson effect provides a very useful technique for maintaining and comparing standards of emf. If we assume the invariance of the voltage–frequency ratio, the emf of a given cell can be monitored very simply in terms of a frequency which of course may be measured to an accuracy of very much better than 1 ppm. In addition, different standards laboratories would be able to check their respective standards against the same reference *in situ*. The

Josephson junction used as a reference has several advantages. It requires only simple experimental techniques and is consequently quite cheap. A laboratory starting from scratch could set up the complete apparatus for perhaps \$25 000 and probably for much less if some basic equipment were on hand.[57] The fact that the voltage–frequency relation does not depend on local environmental conditions, such as gravity and ambient magnetic field, makes the technique ideal for use in different laboratories.

There remains the uncertainty of the calibration of the standard volt in terms of the absolute volt which at present has an uncertainty of about 6 ppm. The $2e/h$ measurement does not directly reduce this uncertainty unless one is prepared to *define* the volt in terms of the Josephson frequency, a step which would require a major revision of the structure of the fundamental constants. However, by using the ratios of the NBS ohm to the absolute ohm and the NBS ampere to the absolute ampere in a fundamental constants adjustment which included $2e/h$ as a stochastic input datum, Taylor, Parker, and Langenberg[58] have been able to reduce this uncertainty to 2.6 ppm (see Sec. VI).

VI. THE ac JOSEPHSON EFFECT, FUNDAMENTAL CONSTANTS, AND QUANTUM ELECTRODYNAMICS

The implications of the $2e/h$ measurements for quantum electrodynamics and the determination of fundamental constants are so intimately related that it is perhaps simpler to discuss them together. These subjects are immensely complicated, and without going into details, we cannot hope to do more than indicate the impact of the new measurements upon them. We shall therefore not give any references to the enormous number of elegant and often very complicated experiments which yield values of fundamental constants, nor to the experiments whose interpretation involves quantum electrodynamics. For a very complete discussion and comprehensive list of references, the reader is referred to the review by Taylor, Parker, and Langenberg.[58] Hughes[59] has also recently given a review of the present status of some of the quantum-electrodynamic experiments, For a discussion on the techniques of quantum electrodynamics, the reader should consult one of the

standard texts.[60] Numerous descriptions of the methods of determining fundamental constants are also available.[61]

VI.1. The Importance and Structure of the Fundamental Constants

There is much more to the importance of the accuracy of the fundamental constants than "just another significant figure." The many and diverse fields of physics produce values of various constants, and a precise comparison of these quantities can yield vital information about the consistency and accuracy of different basic physical theories. Thus, a very accurate measurement of a physical constant in one field may eventually lead to the discovery or removal of an inconsistency in another.

The many experiments capable of producing high-accuracy data often give several different methods of calculating a given fundamental constant. For example, as we shall see in the next section, there are a number of methods of estimating α, the fine-structure constant. The usual procedure in this situation is to perform a least-squares analysis to find the best value of the constant. The data used in this analysis are divided into two groups: auxilliary constants and stochastic input data. The auxiliary constants are known with sufficient accuracy, better than 1 ppm, that they are considered as exact quantities; examples are the velocity of light and the Rydberg constant. The stochastic input data, for example, the Faraday (7 ppm) and the gyromagnetic ratio of the proton in water (4 ppm), are less accurately known. The least-squares analysis is actually performed upon a further set of constants in terms of which each stochastic input datum may be expressed, with the aid of auxiliary constants if necessary. These constants are known as the adjustable constants, and their choice is somewhat arbitrary; examples are Avogadro's number and the electronic charge.

Once all the stochastic input data have been assembled, a critical search is made for discrepant data. It is clearly incorrect to include two or more values for a given quantity which are inconsistent, and a decision has to be made as to which value to retain. To put it another way, it is meaningless to take an average value of two values which differ by several times their standard deviation as at least one value must contain a relatively large systematic error. One tries to find the cause for the discrepancy in a particular datum, for example, in the physical method for measurement used, in the data reduction, or in the correctness of the theoretical interpretation. This may be a difficult or impossible undertaking and one sometimes resorts to an analysis of variance.[12] Values of the adjustable constants are computed using various *subgroups* of the input data in a preliminary least-squares analysis to see which subgroup is most compatible. Upon this basis, one can then select the most reliable data.

One thus finally arrives at a list of stochastic input data which are considered reliable and a final least-squares analysis is performed upon the adjustable constants. A final table of fundamental constants is produced which contains the values of the auxiliary constants and "best" values for the constants used as input data and adjustable constants. Since new experiments and theories are constantly being devised and old ones refined, it is desirable to revise the values of the fundamental constants every few years.

VI.2. Quantum Electrodynamics and the Fine-Structure Constant

Quantum electrodynamics (QED) describes the interactions between charged particles and electromagnetic radiation.[60] The theory has its roots deep in the fundamental axioms of physics, and consequently its ability to describe experimental results to high accuracy is an important test of modern concepts. The following list, by no means exhaustive, gives examples of the application of QED:

(*i*) *Fine structure:* The fine structures of hydrogen, deuterium, and ionized helium (He$^+$) have been a principal source of information on QED. For example, in the $n = 2$ state of hydrogen, the energy levels $2P_{1/2}$, $2S_{1/2}$, and $2P_{3/2}$ are split in zero magnetic field (see Fig. 12). The splitting of the $2P_{1/2}$ and $2P_{3/2}$ levels is the fine-structure interval (ΔE) and is on the order of $\alpha^2 R_y$, about 10 969 MHz, where R_y is the Rydberg constant for infinite mass. The fine-structure splitting is due largely to spin–orbit coupling, which may be calculated from the Dirac theory, but there are QED corrections for the anomalous magnetic moment of the electron and radiative effects of the order of
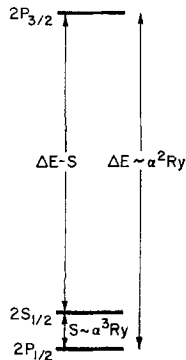
FIG. 12. Energy levels of hydrogen for $n=2$ in zero field. The $2S_{1/2}$–$2P_{1/2}$ splitting is the Lamb shift (S) and the $2P_{1/2}$–$2P_{3/2}$ splitting is the fine structure interval (ΔE).

0.1% and 1 ppm, respectively. The separation of the $2S_{1/2}$ and $2P_{1/2}$ levels is the Lamb shift (S) and is of order $\alpha^3 R_y$, about 1058 MHz. The Lamb shift is completely derived from QED as the Dirac theory does not resolve the $2S_{1/2}$ and $2P_{1/2}$ levels. The theoretical value is given by the product of $\alpha^3 R_y$ with a power series in α, together with small corrections for the size and mass of the proton.

(*ii*) *Anomalous magnetic moments:* According to the Dirac theory, the gyromagnetic ratio (g) of the electron, positron, and muons (μ^+ and μ^-) should be exactly 2. Quantum-electrodynamic calculations predict a radiative correction known as the magnetic moment anomaly a, defined as $a = (g-2)/2$. The corrections are expressed as a power series in α and are on the order of 0.1%.

(*iii*) *Ground state hyperfine splitting:* The interaction between the magnetic moment of the nucleus and that of the electrons causes the energy levels of an atom to split, even in the absence of an applied magnetic field. This effect, known as the hyperfine-structure splitting, has been extensively studied, for example, in the ground states of atomic hydrogen and muonium. The value in hydrogen is approximately 1.42 GHz. The frequency of the hyperfine splitting is known experimentally to extraordinary accuracy, about one part in 10^{12}. The QED corrections arise from vacuum polarizations and again involve α. The correction is on the order of 0.01% (in addition to the g factor correction for the free electron).

All of these QED calculations involve α as an expansion parameter. Clearly, an accurate value is essential for comparing QED predictions with experimental results. In the 1963 least-squares adjustment,[12] there were a number of measurements available for determining α. The final value of the adjustment relied largely upon the fine-

structure splitting in deuterium, measured by adding the value of the Lamb shift (S) to that of the $2S_{1/2}$–$2P_{3/2}$ splitting ($\Delta E - S$). Two other values of α, from the muonium hyperfine splitting and from the g factor for the free electron, were of lower accuracy but essentially in agreement. A fourth value obtained from the hyperfine structure interval in the ground state of atomic hydrogen disagreed with the other values. The value of the hyperfine splitting was known experimentally to incredible accuracy, but there were QED corrections of about 60 ppm. These included corrections arising from the effects of the finite mass, size, and interval structure of the proton which were theoretically rather uncertain at the time. Cohen and DuMond[12] consequently rejected the hyperfine value in favor of the fine-structure value, which was supported by the muonium and g-factor values.

The 1963 value for α thus contained a QED contribution, of the order of 0.1%, which itself involves a knowledge of α. This fact makes the comparison of QED theory with experiments relying upon QED for their interpretation somewhat unreliable since α has, to some extent, been derived by means of this comparison. In addition, it should be pointed out that the fine-structure experiments required the linewidth to be resolved to about one part in 10^3 so that the final value depended critically upon the understanding of the line shape.

The ac Josephson effect measurement is important because the value of $2e/h$ may be combined with other accurately known quantities which do *not* rely on QED to obtain a value of α which (as far as is known) contains no QED corrections whatsoever. These "non-QED" values of α may then be fed into the QED equations to enable a proper comparison with experiments to be made. We shall return to the revised value of α and a comparison of QED with experiments presently.

VI.3. The 1969 Adjustment of the Fundamental Constants

Since the 1963 adjustment was made, new data have become available.[62] Several of the experiments are related to the value of α, and in addition, new theoretical work has been performed on the hyperfine-structure splitting of hydrogen and

on the Lamb shift. Finally, the new value of $2e/h$ from the ac Josephson effect has become available. Since this value is 38 ± 10 ppm smaller than that given by the earlier adjustment, one might expect its influence on the values of some of the constants in the new adjustment to be considerable.

Taylor, Parker, and Langenberg[58] have undertaken a new adjustment of the values of the fundamental constants. Their procedure was similar to that used in the previous adjustments, although they used a different selection of auxiliary constants and stochastic input data. A point of note was their use of electrical conversion factors, the ratio of the NBS ohm to the absolute ohm as an auxiliary constant and the ratio of the absolute ampere to the NBS ampere as a stochastic input datum. As a consequence, they were able to derive a value for the ratio of the NBS volt to the absolute volt with an uncertainty of 2.6 ppm from the least-squares adjustment. This new value has been used to express $2e/h$ in terms of the absolute volt in Sec. III.3. and in Table I.

As has been remarked earlier, the value of $2e/h$ may be combined with other stochastic input data and auxiliary constants to produce values of α^{-1} which do not contain any known QED corrections. Three ways of obtaining indirect non-QED values for α^{-1} are[58]

$$\alpha^{-1} = C_1[\gamma_p^{-1}(2e/h)]^{1/2}, \qquad (6.1)$$

$$\alpha^{-1} = C_2[F^{-1}(2e/h)(\mu_n/\mu_p)]^{1/2}, \qquad (6.2)$$

and

$$\alpha^{-1} = C_3\lambda_c^{-1/2}. \qquad (6.3)$$

C_1, C_2, and C_3 are combinations of auxiliary constants and may be considered exact. γ_p is the gyromagnetic ratio of the proton, F is the Faraday constant, μ_p/μ_n is the magnetic moment of the proton in terms of the nuclear magneton, and λ_c is the electron Compton wavelength.

Taylor et al.[58] performed a least-squares analysis on a set of adjustable constants which included the expressions for α^{-1}. The list of fundamental constants thus produced does not depend on QED and includes a non-QED value of α^{-1} which is 20 ppm smaller than that given by the 1963 adjustment. This value of α^{-1} may then be compared with those obtained from the various QED experiments. The basic finding is that there is fair

agreement with values which are weakly QED dependent (that is, contain QED corrections) but poor agreement with values which are totally QED in origin. Thus, the hydrogen hyperfine splitting is in excellent agreement. This is a very high-precision measurement and any uncertainties involve mostly the theory of proton polarizability which is, in any case, a minor correction. The hyperfine splittings in muonium and positronium are also in agreement but these measurements have much higher experimental uncertainties. The fine-structure splittings in hydrogen and deuterium $(2P_{3/2}-2P_{1/2})$ have been recently remeasured directly and also by combining the $2P_{3/2}-2S_{1/2}$ splitting with the Lamb shift $(2S_{1/2}-2P_{1/2})$. The agreement of the direct measurements is good. Although there is a spread of the indirect values, the agreement is generally acceptable.

We turn now to the quantities which arise totally from QED. The magnetic moment anomalies have relatively high experimental uncertainties; for example the best known one, for the electron (a_e^-), has an assigned error of 26 ppm. The value of α^{-1} obtained from a_e^- is 77 ppm higher than the non-QED value, a discrepancy of some three standard deviations. On the other hand, the Lamb shift values of α^{-1} from both hydrogen and detuerium are significantly *smaller* than the non-QED value, by as much as 280 ± 66 ppm and 400 ± 74 ppm, respectively.

Taylor, Parker, and Langenberg[58] next critically examine these QED values for α^{-1} to see which should be included as data in the final adjustment. There are three preliminary considerations in the selection. First, the QED value must be within two or three standard deviations of the non-QED value. Second, the value must have a sufficiently small error that it contributes a significant weight in the final adjustment; in practice this implies an uncertainty of not more than 6 ppm. Third, any value must be in reasonable agreement with other values obtained by the same method. The anomalous magnetic moment and Lamb shift values were thus eliminated at once. The hydrogen hyperfine splitting was retained, but the other hyperfine values were eliminated as their experimental errors were too high. The fine-structure values (ΔE) were also finally rejected on the grounds that the experimental errors assigned to some of these may have been underestimated.

Table II. A comparison of the values of five constants resulting from the 1969 adjustment with those resulting from the 1963 adjustment (Table taken from Taylor et al.[58]).

Quantity	Symbol	Units	1963 adjustment	Error ppm	1969 adjustment	Error ppm	Change ppm
Inverse of the fine-structure constant	α^{-1}		137.0388	4.4	137.03602	1.5	−20
Electronic charge	e	10^{-19} C	1.60210	12	1.6021917	4.4	+57
Planck's constant	h	10^{-34} J sec	6.62559	24	6.626196	7.6	+91
Electronic mass	m_e	10^{-31} kg	9.10908	14	9.109558	6.0	+52
Avogadro's number	N	10^{26} kmole^{-1}	6.02252	15	6.022169	6.6	−58

This last omission did not, in fact, change the final value of α^{-1} by a very great amount. Thus, the only values of α^{-1} included in the final adjustment were the non-QED value and hydrogen hyperfine-structure value.

The impact of the new value of α^{-1} on the other fundamental constants can be seen from Table II in which we list the 1963 and 1969 values of five constants. We see that the change in the value of each constant is considerable, in all cases several standard deviations.

VI.4. Discussion

The agreement between the Josephson and hydrogen hyperfine-splitting values of α^{-1} is excellent. Both of these measurements are of high precision. This agreement between two totally different types of experiments is very convincing and provides the best *independent* verification of the accuracy of the ac Josephson voltage–frequency relation. The fine structure value (from the $2P_{3/2}$–$2P_{1/2}$ transition) in hydrogen and deuterium is less accurate but in acceptable agreement with the other two; this value has only small QED corrections. On the other hand, the g-factor anomalies and Lamb shift, which depend completely on QED, do seem to be in significant disagreement. Of the magnetic moment anomalies, only that for the electron is badly in error, but in any case there is only one high-precision value available at present. More experimental data are required before one can draw any real conclusions. There are several Lamb shift measurements available, all disagreeing with the theory in the same direction and by comparable amounts. Thus either there is an unsuspected systematic error in the experiments, or there is something wrong with the theory. There is considerable activity in both theory and experiment at present, and one may hope that the situation will become clarified in the reasonably near future.

VII. CONCLUSIONS

The experiments performed by Parker, Langenberg, Denenstein, and Taylor[11] on the ac Josephson effect have yielded a value of $2e/h$ which is 38 ± 10 ppm smaller than that given by the 1963 least-squares adjustment.[12] An independent measurement by Petley and Morris[45] has confirmed this result. There are excellent theoretical grounds for believing the Josephson voltage–frequency relation to be exact, at least at the level of 1 ppm and probably at a much higher level. The invariance of the value of $2e/h$ under all experimental conditions strongly suggests that the experiments do indeed measure a fundamental physical quantity.

The impact of the measurement has been three-fold. First, the Josephson junction seems likely to provide the most convenient and satisfactory means of comparing and maintaining standards of electromotive force.[57] Second, it has played a prominent role in a new adjustment of the fundamental constants.[58] The new values of some of the constants differ markedly from those of the previous adjustment. Third, the measurement of $2e/h$ has provided for the first time a method of obtaining a high-precision value of the fine-structure constant without invoking the aid of quantum electrodynamics.[58] This value of α is in convincing agreement with that obtained from the hydrogen hyperfine-splitting experiments. Good agreement has also been obtained with the fine-structure splittings in hydrogen and deuterium which involve only small quantum electrodynamic corrections. On the other hand, the Lamb shift and magnetic moment anomalies, which are totally quantum electrodynamic in origin, yield values of α which differ significantly from the

Josephson effect value. It appears at present that either something is amiss in the theory or alternatively, that there is a systematic error in the experiments on Lamb shift and magnetic moment anomalies. More work is required to clarify this situation.

Finally, the experiments on Josephson junctions are a very striking illustration of the macroscopic quantum phase coherence in superconductors. The invariance of the frequency–voltage ratio convincingly demonstrates that the junction is a very simple quantum system while the sharpness of the induced steps indicates that the phase difference between the two superconductors is an extremely well defined quantity.

ACKNOWLEDGMENTS

Notes added in proof: Since this paper was submitted for publication, there have been several relevant developments. We briefly list them here:

(i) T. Appelquist and S. J. Brodsky [Phys. Rev. Letters 24, 562 (1970)] have recalculated the fourth-order electrodynamic corrections to the Lamb shift using computer techniques. Their result disagrees significantly with previous values. Using the non-QED value of α, the new theory agrees with most of the experimental values of the Lamb shift to within one standard deviation. Although it is important to check this new calculation by independent methods, it seems likely that the discrepancy between theory and experiment has been removed.

(ii) B. W. Petley and K. Morris (to be published in Metrologia) quote their final result for e/h as 0.3 ppm greater than their preliminary value reported in Sec. III.4.

(iii) A. Denenstein, T. F. Finnegan, D. N. Langenberg, W. H. Parker, and B. N. Taylor (to be published in Phys. Rev. B, 1 June 1970) have repeated the e/h measurement using essentially the same equipment as in the determination by Parker et al.[11] The new result is 0.6 ppm higher than that of Parker et al.[11] Denenstein et al. have also discovered the *possibility* of a small systematic error in the value of Parker et al. However, this error, if present at all, was less than one standard deviation.

(iv) T. F. Finnegan, A. Denenstein, and D. N. Langenberg [Phys. Rev. Letters 24, 738 (1970)] have used new experimental techniques to determine e/h to higher precision. Their value is 483.59365 ($\pm$0.46 ppm) MHz/μV$_{\mathrm{NBS69}}$. This result is 0.3 ppm higher than that of Parker et al.[11] The authors feel that their experimental accuracy can be improved to the level of 0.1 ppm in the near future.

(v) V. E. Kose and D. B. Sullivan [J. Appl. Phys. 41, 169 (1970)] have studied the microwave-induced steps on the i–v characteristic of a point-contact junction in the presence of noise. They used both voltage- and current-noise sources. They found that in each case, the noise did not affect the voltage at the midpoint of the induced step (see Sec. IV.5).

(vi) T. A. Fulton and R. C. Dynes ("Interpretation of Quantum Interference Properties of Double Josephson Junctions," paper presented at the 1970 Applied Superconductivity Conference held at Boulder, Colorado, June 1970 and to appear in the conference proceedings) have investigated the current–phase relation of various types of superconducting junctions. They found that the relation $j=j_1\sin\Delta\phi$ [Eq. (2.2)] was valid irrespective of the type of junction provided that the junction was sufficiently weak. However, if the junctions are strongly coupled so that flux flow may occur, the sinusoidal behavior is unlikely.

(vii) F. Bloch (to be published in Phys. Rev. B, 1 July 1970) has given a new derivation of the Josephson voltage–frequency relation. The treatment emphasizes the fundamental nature of the effect and the exactness to be expected of the relation.

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Fresnel Diffraction Using a He–Ne Gas Laser

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An advanced laboratory experiment of Fresnel diffraction is presented. A He–Ne gas laser is used as the source and a wire is used as the opaque diffracting strip. A photograph of the diffraction pattern is compared with the intensity diagram predicted by the Cornu spiral method. Agreement is clear and impressive, although minor differences are detectable.

INTRODUCTION

In the March 1969 issue of the American Journal of Physics, Malacara, Berriel, and Rizo[1] discussed the construction of helium–neon gas lasers at 6328 Å. This article presents an advanced laboratory experiment using a He–Ne gas laser as a device for the study of Fresnel diffraction of an opaque strip.

In classical optics, the diffraction pattern of a narrow opaque strip parallel to the axis of a cylindrical wave can be analyzed by the Fresnel method. One can use the Cornu spiral for predicting what the pattern will be like. Since the Cornu spiral is presented in most optics courses but rarely used in laboratory experiments, the following experiment should be of interest. Of course, one reason such experiments have been rare has been the difficulty of obtaining a strongly coherent source of light. With lasers readily available, that reason is no longer valid for many laboratories.